Acknowledgments

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Effect of Geometric and Material Nonlinearities on the Propellant Grains Stress Analysis

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Nomenclature

a	= inner radius
$[B]$	= matrix relating strains and nodal displacements
b	= outer radius
$[D(\bar{\sigma})]$	= elasticity matrix
E_c	= Young's modulus for metallic casing
E_1,E_2	= Young's moduli for FRP casing material in axial and hoop directions, respectively
G,G_{12}	= shear moduli for propellant and FRP casing, respectively
h	= thickness of casing
K	= bulk modulus
p	= internal pressure
R	= radial coordinate
u	= radial displacement
w	= axial displacement
Z	= axial coordinate
ν_c,ν_1,ν_2	= Poisson's ratio for metal and FRP casing, respectively
$\epsilon_z, \epsilon_\theta$	= axial and hoop strains, respectively
$\bar{\sigma}$	= mean normal stress

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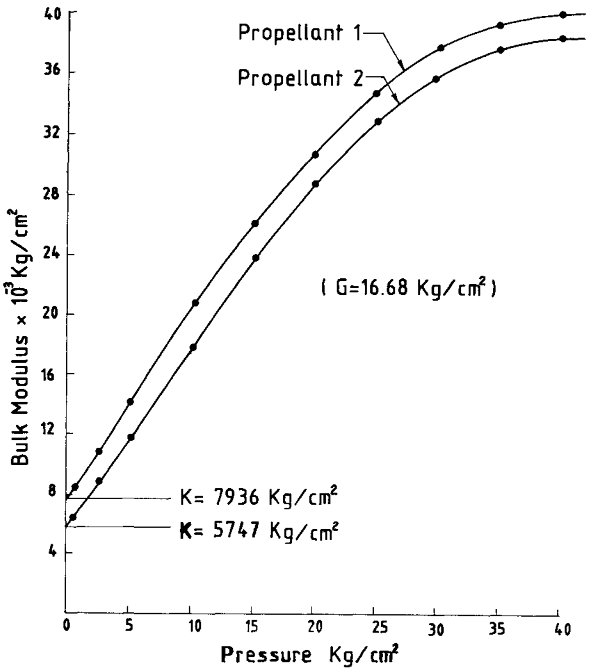

Fig. 1 Variation of bulk modulus with pressure for two HTPB-based propellants.

Introduction

THE presence of porosity or internal voids in solid propellant grain material affects the volumetric response significantly when subjected to pressure load. In a bulk modulus measurement experiment when the propellant specimen is subjected to hydrostatic pressure, the response is highly nonlinear[1] at lower pressure because most of the energy is going into collapsing the internal voids. In this Note, the effect of such material nonlinearity and the geometric nonlinearity on the elastic stress analysis of pressurized propellant grains is studied.

The variation of bulk modulus with hydrostatic pressure obtained from two HTPB-based propellants is shown in Fig. 1. Bulk modulus variation for propellant 1 is used for the analysis of a simple case-bonded cylindrical grain, whereas that for propellant 2 is used for the analysis of an axisymmetric slotted grain. The studies indicate that the effect of material nonlinearity is more predominant as compared to the effect of geometric nonlinearity.

Incremental Finite-Element Analysis

A simple axisymmetric triangular ring element is used to derive element stiffness matrices and load vectors. The effects of geometric and material nonlinearities in the stress analysis have been handled here by the widely used incremental (or step-by-step)[2,3] method. The effect of geometric nonlinearity is incorporated into this formulation by modifying the $[B]$ matrix at each load step, taking into account the change in grain geometry. The nodal forces on the inner surface are modified at each load step considering the expansion of the inner surface. To take into account material nonlinearity, the elasticity matrix

$$[D(\bar{\sigma})] = \begin{bmatrix} K(\bar{\sigma})+\frac{4}{3}G & K(\bar{\sigma})-\frac{2}{3}G & K(\bar{\sigma})-\frac{2}{3}G & 0 \\ & K(\bar{\sigma})+\frac{4}{3}G & K(\bar{\sigma})-\frac{2}{3}G & 0 \\ & & K(\bar{\sigma})+\frac{4}{3}G & 0 \\ \text{(Symmetry)} & & & G \end{bmatrix}$$

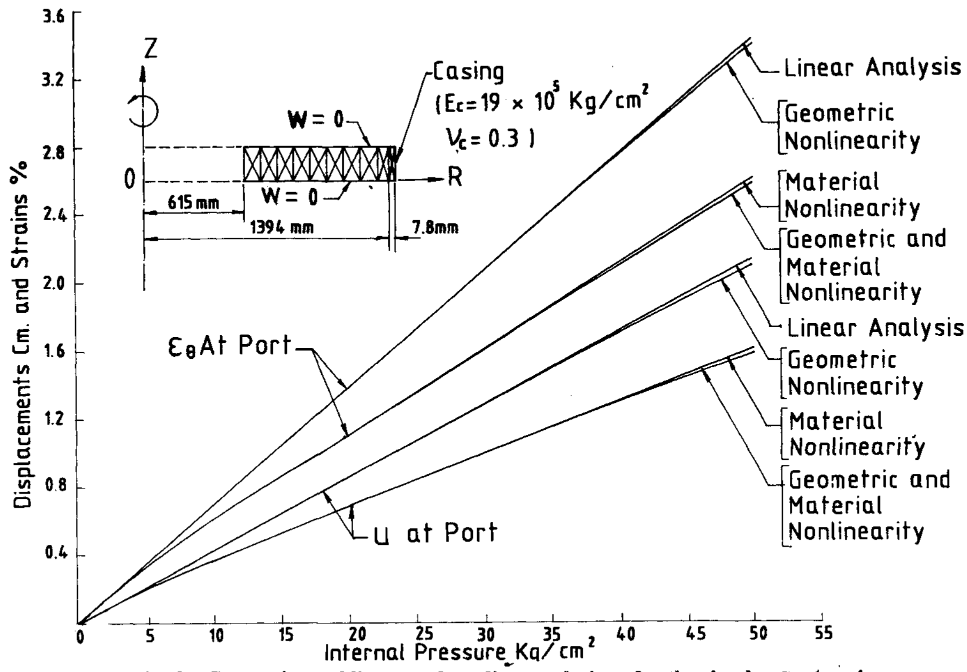

Fig. 2 Comparison of linear and nonlinear solutions for the circular port grain.

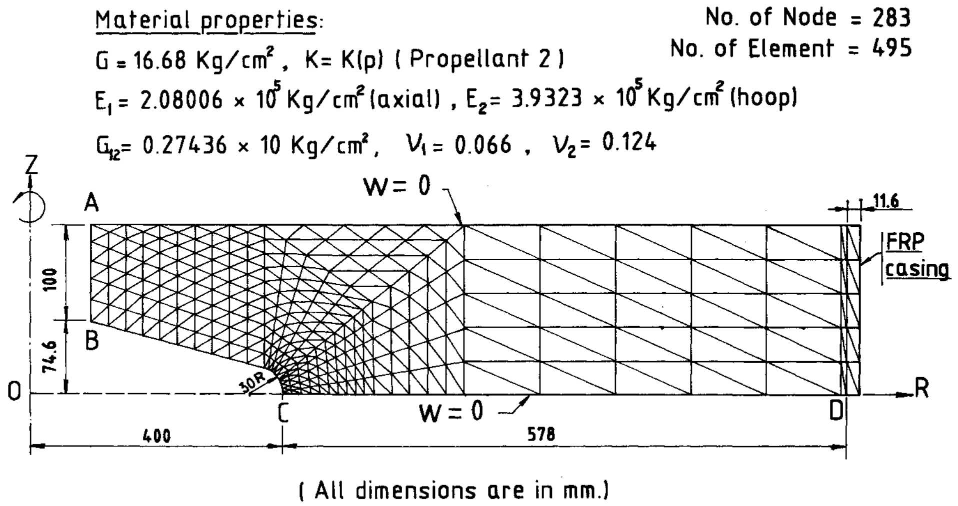

Fig. 3 Finite-element idealization of an axisymmetric slotted grain.

is recalculated at each load step by considering an appropriate bulk modulus value. First, the bulk modulus is calculated based on the total incremental internal pressure; later, the value is modified iteratively as a function of total incremental mean normal stress at the element centroid. The total incremental quantities are obtained by summing the incremental quantities at each load step. The shear modulus G, which governs the response of a material to a change of shape, is assumed to be constant with respect to the change in hydrostatic pressure. This is based on the studies presented in the literature[1,4] for nearly incompressible rubbery material.

Results and Discussion

Thick Case-Bonded Cylindrical Grain

First, a simple, case-bonded, infinitely long, cylindrical grain with a circular port has been considered. A thin slice of the grain is idealized by the axisymmetric triangular ring element with appropriate boundary conditions as shown in Fig. 2. Nonlinear material properties of propellant 1 (see Fig. 1) are used for this problem. Linear analysis is carried out considering an initial bulk modulus of 7936 kg/cm^2 (778.2 MPa). Convergence of the solutions from nonlinear analysis was studied and it was found that results obtained with the load step of $\Delta p = 1.0$ kg/cm^2 (0.098 MPa) were accurate enough. All the nonlinear analysis results presented here are obtained by considering $\Delta p = 1.0$ kg/cm^2 (0.098 MPa).

The effects of pressure on the linear and nonlinear analysis results are compared in Fig. 2. Results show that the radial displacement and hoop strain at the port surface obtained by using nonlinear finite-element analysis considering geometric nonlinearity, material nonlinearity, and combined geometric and material nonlinearities are 1.3, 25.4, and 25.9% lower than those obtained by using linear finite-element analysis. This indicates that, for a simple circular port grain geometry, the effect of geometric nonlinearity is negligible, whereas the combined effect of both of the nonlinearities is mainly influenced by the material nonlinearity.

Axisymmetric Slotted Grain

An axisymmetric, long case-bonded solid propellant grain[5] with a number of circumferential slots equally spaced at the inner surface of the grain subjected to an intenal pressure load is considered. Finite-element idealization of one-half of a slot

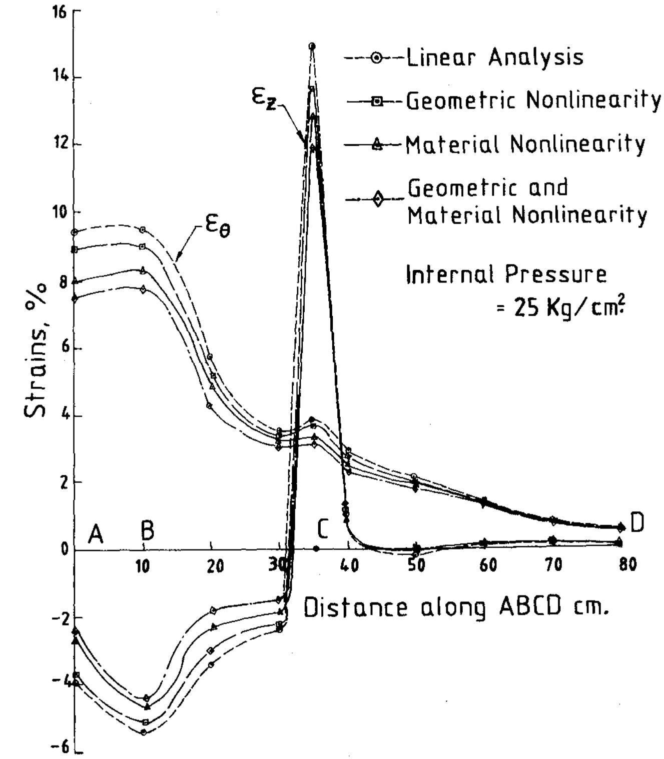

Fig. 4 Variations of axial and hoop strains along ABCD obtained from linear and nonlinear analyses.

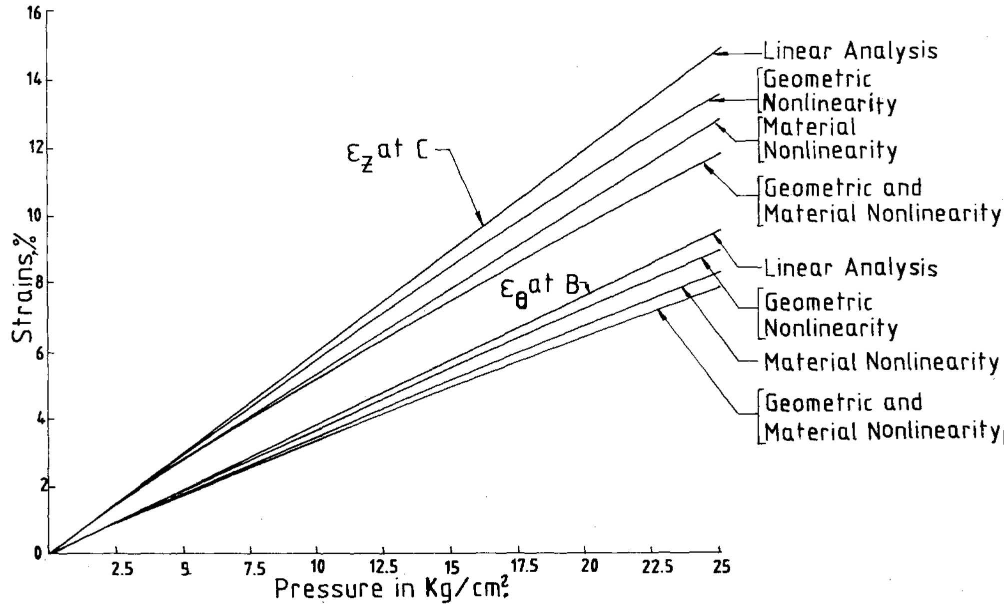

Fig. 5 Comparison of linear and nonlinear solutions for the slotted grain.

and the material properties used in the analysis are given in Fig. 3. Bulk modulus variation with internal pressure for the HTPB-based propellant 2 (see Fig. 1) has been used in the analysis for material nonlinearity; for linear analysis, the initial bulk modulus value of 5747 kg/cm^2 (563.55 MPa) has been used.

Variations of axial and hoop strains obtained from linear and nonlinear analyses are plotted in Fig. 4 along the line ABCD (see Fig. 3); AB corresponds to the circular port portion, C is the slot tip, and CD corresponds to the web thickness. Figure 4 shows that the maximum hoop strain at the circular port portion obtained from linear and nonlinear analyses with geometric, material, and the combined geometric and material nonlinearities are 9.5 (linear analysis), 8.9, 8.3, and 7.8%, respectively. The corresponding values for the maximum axial strain at the slot tip are 14.9 (linear analysis), 13.6, 12.8, and 11.8%, respectively. The effect of pressure on the maximum axial and hoop strains obtained from linear and nonlinear analyses is presented in Fig. 5. The results presented here indicate that, for a slotted grain configuration, the effect of both geometric and material nonlinearities on the maximum tensile strains at the port surface is considerable, although the influence of material nonlinearity is more predominant.

Conclusions

For propellant grains with a complicated inner port configuration similar to stars or slots, the geometric nonlinearity has considerable influence in the computation of maximum tensile strain at the port surface. The material nonlinearity reduces the maximum tensile strain quite considerably at the inner surface of the grain. The effect of material nonlinearity is seen to be more predominant as compared to the effect of geometric nonlinearity.

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